Conducting an Online Inquiry-Based Mathematics Workshop: A First Experience

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its publication of the *Standards* in 1989 and 2000, the National Council of Teachers of Mathematics has taken a stance that mathematics instruction should be an active experience for the mathematics student. NCTM (2000) states that students should experience problem solving and be given the opportunity to collaborate and communicate with classmates. "Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections" (*Principles and Standards Executive Summary*, p. 4). At the same time, many states have enabled and supported the development of coursework available completely online. Cyber-schools with cyber-classes afford students the opportunity to complete courses anytime, anywhere, without the restrictions that come with commuting to the school or classroom.

Can the ideal standards-based mathematics classroom and the convenient cyber-classroom co-exist? As a first step to answering this question each of four mathematics professors developed an online mathematics module consisting of a set of inquiry-based activities to be completed entirely online. The modules were designed in such a way that the students would need to collaborate with each other to complete the lesson objective. Each module was self-contained. The students had all the necessary materials they needed to complete each activity. Each instructor had an online presence in his or her course component and interacted with the students throughout the activities. The course was offered as a four-week online graduate workshop for inservice mathematics teachers. The first two weeks of the workshop were devoted to geometry and history concurrently, and the last two weeks to statistics and abstract algebra. Five days before the start of the workshop, students were asked to complete an online tutorial of WebCT, the online course management system.

Related Literature

Online curricula, including online mathematics curricula, have been increasing in number in the last decade. Within the World Lecture Hall database of online courses at universities world-wide, there were, as of the date of this writing, fourteen mathematics courses listed. In the "OhioLearns!" Catalog of online courses at Ohio universities, there were forty-one courses within the mathematics category. Some online material has been developed as supplements to other modalities, such as the online mathematics curriculum Virginia Tech has developed as part of their Math Emporium, a technology-assisted learning community (Moore, 2001). Developing online curriculum is, in itself, challenging. Yet, developing online

mathematics coursework presents extra challenges, such as being at least as complete as a textbook since mathematics textbooks are not necessarily written for independent study. Further, online mathematics courses must have clear mathematical typography since students are not accustomed to reading mathematics in varying forms (Allen, 2001). Testone (2003), through experience as both an online mathematics student and online mathematics instructor, found that an online instructor has a great impact on the quality of a student's online experience and that the instructor needs to develop assignments that are conducive to online learning and student interaction.

In scanning the list of online courses found in the OhioLearns! Catalog, the vast majority were lower-level mathematics courses such as basic mathematics and consumer mathematics. The materials in those courses are primarily designed to be done independently, not collaboratively with classmates.

While there has been a great deal of development in online curricula in the past decade, online learning that is constructivist or inquiry-based is more difficult to find. Both the National Council of Teachers of Mathematics (2000) and the National Research Council (2002) endorse science and mathematics curricula that encourage students to learn by inquiry and discovery-based methods. Yet, the constructivist approach to an online course can be challenging (Lewis, Spector, & Burkett, 2001). In the constructivist model, the instructor gives up the desire to transmit information to the students, and the students are encouraged to construct meaning for themselves.

Cooperative learning, as a teaching technique, has been found to promote higher levels of achievement, productivity, and motivation (Johnson & Johnson, 1989). Graham (2002) discusses group structures in online learning environments and the impact that virtual environments have on these groups. Creating groups, structuring learning activities, and facilitating group interactions are considered important factors for promoting effective group work. Online group tasks have been found to take longer than the same tasks done face-to-face. Also, in the virtual environment, lack of nonverbal cues and an absence of social influences by group members may depersonalize the tasks. On the other hand, the absence of these interpersonal communication tools in the virtual environment may also alleviate the instance of social pressure exerted by one group member over others.

Lesley University in collaboration with TERC, an organization dedicated to conducting research on and creating resources to support innovative teaching in mathematics and science, created an online master's degree in inquiry-based science education. They then conducted research on an online teacher group in comparison with a face-to-face control group. The preliminary results of their research show that online teacher participants reflected more on the learning process on their own, and that they made more explicit reference to recognizing the value of collaborative learning than their face-to-face counterparts. The online participants also showed statistically more significant gains in post-course science concept understanding and in post-course confidence in teaching students to use and develop various inquiry skills. Further, the online learners showed as much incidence of social interaction as the on-campus learners (http://scienceonline.terc.edu/summer_2002_findings.html).

The Online Curricula

The workshop in the Ohio's Online Inquiry-Based Mathematics project consisted of four modules, each based on a particular mathematics content area: geometry, history of mathematics, abstract algebra, and statistics. Each of these four represent curricula typically found in the content preparation of mathematics teachers.

All students in the course accessed the material through the same web-based portal, WebCT, the university's adopted online course management system. Once in the course, the students found four modules, each designed by a separate instructor. All the modules had common technological resources

that WebCT provided, such as discussion boards and white board, but the extent to which each student used the resources within each module depended on the design of the module activities.

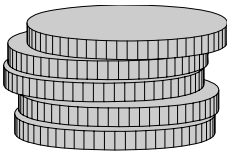
Geometry

In the geometry component, students completed an initial set of four introductory lessons to become familiar with the use of the dynamic geometry software *Cabri Geometry*. Using *Cabri* on a local network is very slow, and that problem is magnified when one is dialing into the network. Fortunately, the students were provided with a trial version of *Cabri* that they could install on their own computers for the duration of the workshop. After that, they completed three of a set of four inquiry-based lessons on traditional Euclidean geometry and two of a set of three more inquiry-based lessons on modern Euclidean geometry. The seven inquiry-based lessons had been developed using *Cabri Geometry*. They were previously used in a face-to-face setting, some in an upper-level geometry course and others in a "Mathematics for Elementary Teachers" course. For the activities, students grouped in teams, communicating with their teammates using the (asynchronous) threaded discussion facility of WebCT.

Statistics

The statistics component of this workshop included a variety of web-based inquiry activities. Several of the activities focused on the core ideas of statistical inference. The students communicated with their teammates using the (asynchronous) threaded discussion facility of WebCT. Topics and activities were not presented in a one-at-a-time succession, as would be done in a classroom setting. Instead, students often worked simultaneously on activities pertaining to two or three different topics. This allowed more topics to be covered, and removed some of the "dead time" in the asynchronous communication among teammates; while waiting for a teammate's input on one activity, a student could use the time on a different activity. In addition, students worked in teams to design and carry out a project involving data collection and analysis. The chat room in WebCT was utilized for brainstorming initial project ideas, and for discussions regarding the project data analysis.

One of the activities was designed to cause the students to reflect on the logic behind statistical hypothesis testing by considering a claim that pennies, when balanced on edge, are more likely to fall heads up than tails, when the surface is lightly tapped. Versions of this activity had been done previously by the instructor as a classroom demonstration, coupled with a related homework assignment. For this workshop the activity was separated into three parts, with aspects of discovery and cooperative learning included. In Phase I, the students were to consider what results would constitute convincing evidence that the claim is true; if an experiment involving 20 pennies were to be performed. The students shared their opinions and reasoning via the discussion facility of WebCT. In Phase II, the students were given some review material pertaining to the binomial distribution, and directed to rethink their answers to Phase I in light of the relevant binomial probabilities. Some of the notions of hypothesis testing, such as *null* and *alternative hypothesis*, were introduced at this time in the instructions to Phase II. Again, relevant discussions took place online using WebCT. By Phase III the students had each conducted the experiment themselves, and they drew conclusions based on their results. The instructor interacted with the students throughout the three phases of this activity by intervening as needed in the discussions and adjusting the instructions for each Phase. In the process, students were led to think through and discover such notions as p-value, significance level, and comparison of observed counts to expected counts based on a null hypothesis.



History of Mathematics

In the history of mathematics component of the workshop, the students were required to read excerpts from *The Geometry of Rene Descartes* and Fermat's *On Analytic Geometry*. The students were given a list of questions about each excerpt designed to help them discover the different approaches each author took to develop his version of analytic geometry. The last assignment for the students was to synthesize

and compare the two approaches. They all were able to give the correct conclusion: Descartes started with curves and derived equations while Fermat started with equations and sketched the curve associated with them. All the groups used threaded discussions for their interactions. One group chose to make significant use of the chat room while another decided to use the white board extensively.

Abstract Algebra
The objective of the abstract algebra component was to develop the notion of an algebraic group by exploring symmetric transformations of the plane. Four sets of paper manipulatives were sent to each student via postal mail prior to the start of this component. Each team studied a different figure and shared the results with the other teams, as all the results were needed for subsequent activities.

The general course syllabus included course policies, expectations and requirements, assessment and grading guidelines (rubric given), and technical support information that was common to all the components. Moreover, each component had an individual syllabus with more specific requirements, guidelines, and component content.

The instructors stated that the two-credit workshop would require a minimum time investment of 90 hours for the workshop content and whatever additional time it would take the student to become acquainted with the course management software (WebCT). The instructors added that inquiry-based learning is a primary focus of the workshop and that this is best accomplished through teamwork. Consequently, the students were expected to provide significant contributions to their teams' assignments/activities. Contributions were to be assessed and would be a major part of the course grade. Each instructor assessed the students in their module and then collaborated with the other instructors to determine a pass/not pass grade for the workshop. The rubric that was used by each instructor to evaluate individuals and teams is provided in Figure 1.

Rubric to evaluate team submission of assignment:					
Characteristics of team submission	**Likert Scale: __1_____2_____3_____4_____5____ Poor Below Average Average Above average Excellent**				
Completeness					
Correctness					
Clarity, Precision, Depth					
Bonus (Suggested Extensions)					

***Rubric to evaluate individual's participation in assignment*:**				
Participation	**Likert Scale: __1_____2_____3_____4_____5____ Poor Below Average Average Above average Excellent**			
Initiative				
Key contributions				
Questions posed/ responses given				
Clarity, Precision, depth of comments				

Figure 1: Assessment Rubric

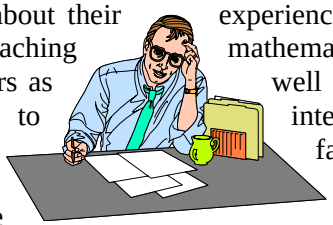
The Results

There were ten graduate students who began the four-week summer workshop, but four students withdrew after the first week. Questionnaires were given out at the beginning and end of the workshop. The questionnaire given at the beginning was designed to assess the students' familiarity with WebCT and different software used, as well as their previous experiences with inquiry-based learning. The questionnaire given at the end asked the students for feedback on the workshop.

The ten original participants widely ranged in years of teaching experience with half reporting being very comfortable with technology. Within the first week, many students felt overwhelmed by the expectations of the workshop, but those who remained (six students) appeared to adjust to the unique nature of trying to work within groups while communicating using only the computer.

The students stated that the disadvantages of the course experience were the inability to communicate in a timely and efficient manner and the difficulty of easily sharing a word document (or Minitab document, etc.). The need to learn WebCT as well as mathematics-specific software within the given timeframe proved to be a burden to some of the students. Some complained that the workload of the course was too heavy; and a few felt that the material was not useful for the classroom.

The students cited several advantages to the online workshop. Most enjoyed the flexibility of being able to work at home, the variety of topics, and the opportunity to learn about new software programs and their integration in mathematics. They noticed that in an online environment the quiet student always has a chance to speak and teachers can monitor ALL students' involvement and comprehension. They liked the paper trail of conversations to jog their memory.



The instructors also filled out a questionnaire about their experience with the workshop. Regarding communications issues in teaching mathematics online, the instructors emphasized that challenges existed for instructors as well as for students. For instance, the instructors needed extra time to interpret and refine students' writings. They noted that the lack of face-to-face contact and nonverbal indicators required them to be more precise and to wait for students to indicate when they did not comprehend or were confused. The instructors also found the chat room useful for establishing a feeling of connection in a more personal way.

Student contributions to the asynchronous threaded discussions were generally better thought out and to the point than in the live chat room. Students who were more familiar with instant messaging seemed to readily embrace the chat room discussions, as compared to those who were not accustomed to instant messaging. Even with teams of only three, synchronous discussions were difficult to schedule. Schedule conflicts also impeded progress in the asynchronous discussions. Therefore, we recommend that student schedules be considered when forming teams.

Instructors also noticed effects related to time. They found it time-consuming to compose and post comments as well as to allow the students to take the lead and react accordingly. They found it difficult to determine the pacing for the assignments. The instructors reported that they spent an average of three hours a day online, in addition to offline and pre-workshop preparations.

When asked what they would do differently if they were to do it again, the instructors responded that if the targeted population stayed the same, i.e., in-service teachers who are relatively unfamiliar with the current technology used in college, they would restructure the beginning of the workshop to allow students more time to become comfortable with the technology. They hope that in the near future WebCT would offer more technological capabilities, such as shared workspace and video-conferencing, as well as the ability to communicate using mathematical notation and graphs. Secondly, they would add individual

assessments of the students' content knowledge. Also, the instructors believed that the amount of content that was placed in the workshop was somewhat more than what would be expected in a two-credit hour workshop. The instructors did not anticipate that four modules would be overwhelming, but they did not consider the added factors of the stress on the students of learning the new software as well as experiencing delays in communicating with each other. The students' lack of experience with inquiry-oriented activities added additional stress.

When teaching inquiry online, the instructors learned that they needed to differentiate between cooperative learning challenges and online challenges. They also decided that in the future, they would group students based on scheduling availability and would implement strategies that would help develop collaborative habits in the students. The instructors noted that the students' background in inquiry-based learning impacted their attitude and performance in the workshop, in that those with more experience in face-to-face inquiry-based learning were better able to transfer this approach to the online environment.

All the instructors responded that they would be willing to teach online inquiry-based mathematics again, but only under certain conditions. All agreed that WebCT is a useful pedagogical tool. They concluded that mathematics inquiry online is possible, but has drawbacks. While it can't be done as effectively as in the face-to-face setting, some aspects are better, such as being able to eavesdrop on every conversation.

Conclusions

So, is online inquiry mathematics possible? Our conclusion is "yes" (the instructors had written evidence of the "aha" moments), but it is difficult. Some of the factors that could help facilitate this learning environment could be better technology for shared mathematics, such as tablet-writing tools that can be shared in real time. In order for the learning to be effective, students need to become accustomed to doing inquiry in all learning systems, not just online. Instructors that undertake this method need to know that a heavy investment of instructor time is required. Online learning is time-consuming to develop; inquiry-based learning is time-consuming to develop; online inquiry-based learning is very time-consuming and requires a good deal of instructor creativity. However, the flexibility it offers students with regard to time and learning environments, and the opportunity to engage in the higher order thinking skills, may make this instructional venue appealing to many faculty and students.

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Quote:

Stimulus processing that does not reach awareness in the form of conscious content can nevertheless be stored implicitly or unconsciously and have important influences on thought and behavior at some later time.

DeLoux, J. (1996). *The Emotional Brain*, 33. Simon & Schuster Paperbacks. New York.

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